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Irregular, Non-linear Vibration in the System with an Elastomer Element

Jarosław Dyk, Jerzy Osiński

Warsaw University of Technology
jaroslaw.dyk@ipbm.simr.pw.edu.pl

Abstract

The considered nonlinear system with an elastomer element is subjected to the harmonic excitation and undamped practically. The performed analysis confirms that there is possibility of irregular motion in the system, similarly as in reality chaotic motion in case of conservative and Hamiltonians systems. Such kind of motion seems to be specific to the non-linearity of elastomers for heavy loaded systems that are forced at the frequencies close to ones from the resonant zones.

Keywords: irregular vibration, elastomer, numerical solutions

1. Introduction

As a result of the research on the oscillation systems with elastomer elements, how it has been reported among others in the work [Boczkowska A., at al, 2008], an irregular character of obtained time histories of vibration is revealed. Therefore, the question is: weather the specific, strongly non-linear characteristic of an elastomer elasticity may be the reason of the chaotic motion and irregular solutions of equations.

Elastomers for typical conditions of use are highly non-compressible materials that possess extremely low level of damping (which can be neglected in practice) and very high (up to 1000 %) convertible elastic deformations [Ward M., 1971]. Such materials are described by the complex well-developed theory of the hyperelastisty, and that is why in practice the Mooney - Rivlin, Ogden or Yeoh models [Ward M., 1971] are often utilized.

Our aim in this report is to analyze the numerical results of computation for forced undamped vibration (and also in order to emphasise some problems - considering damping) in the system with an elastomer element as well as to evaluate the possible irregular motion.

2. Equation of motion

We start with a description of the structure characteristics. The spatial model consist of the mass *m*, weightless elastic constraints which imitate elastomer material and weightless viscoelastic damper with the linear characteristic. Properties of elastomers are described by the classical Mooney's model. The functional of elasticity is wrote down by the formula (the notation like in the monograph [Ward M., 1971]):

$$U = C_1(I_1 - 3), \tag{1}$$

where: I_1 is the first invariant of the state of strain and it is described by

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \tag{2}$$

In this expression $\lambda_1^2, \lambda_2^2, \lambda_3^2$ are the relative elongations and are defined by use of the finite deformation theory [Ward M., 1971]. We take advantage of the condition of incompressibility expressed by the formula

$$\lambda_1^2 \lambda_2^2 \lambda_3^2 = 1, \tag{3}$$

we obtain

$$\lambda_2^2 = \frac{1}{\lambda_1}; \ \lambda_3^2 = \frac{1}{\lambda_1}. \tag{4}$$

After the substitution (2) and (4) into (1) we obtain

$$U = C_1\left(\lambda_1^2 + \frac{2}{\lambda_1} - 3\right). \tag{5}$$

Now, we differentiate the above formulated expression according to

$$\sigma_1 = \frac{\vartheta U}{\vartheta \lambda_1} \tag{6}$$

and introduce the dimensional co-ordinate *x* by use of the approximate expression accordingly:

$$\lambda_1 = \varepsilon_1 + 1 \tag{7}$$

or

$$\lambda_1 = \frac{x+l}{l}$$

(l is the length of the elastic element), and then we obtain the expression describing the elastic force:

$$F = k\left[x + l - \frac{l^3}{(x+l)^2}\right], \tag{8}$$

where the stiffness coefficient is expressed by $k = \dfrac{2C_1}{l}$.

We assume that the object of our research is an elastomer EPUNIT© that has been worked out at Faculty of Material Engineering of the Warsaw University of Technology. This material was experimentally examined and the results were presented in [Boczkowska A., at al, 2008]. The results of the experiments for this elastomer presented in the monograph [Żach, P., 2013] are also taken into consideration.

It has already been emphasized that this elastomer is strongly non-linear, thus the governing equation of motion is in the form

$$m\ddot{x} + c\dot{x} + k\left[x + l - \frac{l^3}{(x+l)^2}\right] = P(t), \tag{9}$$

where: m – mass, c – damping, k –spring constant. In the subsequent computation there have been accepted the values of parameters as follows: m =0,1 kg, k = 9034 N/m and l = 0,01 m. Evaluation of damping forces has been conducted by use of the non-dimensional theoretical damping factor $\zeta = c/[2(m\,k)^{1/2}]$.

The systems with elastomers are mainly used as shock absorbers. For this type of devices it is obvious that forces act rather inside of compressing range of loads. Thus, for this kind of loads the appropriate expression describing input forces is formulated in the form

$$P(t) = F\sin\nu t - F_{\mathbf{0}} \tag{10}$$

where, ν - frequency of acting variable force, F- amplitude of this force, F_0- constant load.

3. Vibration of the heavy – loaded system with an elastomer element

As we have just mentioned, the irregular vibration had been observed in the systems with the elastomer element. Thus, it is reasonably to research into the nature of this phenomenon. As an introduction to the detailed study of the subject we focus on the model described by the equation (9) and its solutions obtained on the basis of computation.

On this stage of the research, mainly the fundamental methods of solving and analyzing the complex dynamic processes described by the differential equations have been chosen. The solutions of the differential equation have been finally obtained using the Gear method of the numeric integration. To obtain the sufficient accuracy of computation for long time periods we have taken advantage of methodology similar to the one applied in studies [Warmiński J., 2010, Pascal M., 2012, Chin C.M., Nayfeh A.H., 1997] and in previous studies of author's [Dyk J., at al, 1994]. Taking into consideration the question what kind of the motion is it, we are convinced that only a thorough examination in every respect by use of the sufficiently wide range of dynamic factors help us to detect the properties of dynamic signals. We use some of these factors for our purposes, and therefore we focus on the analysis of the results obtained i.a. on the basis of: statistical evaluation of data, frequency spectrum and the Poincaré portraits of the space trajectory.

4. Numerical calculation

Non-linearity of the equation (9) is illustrated by the elastic force plot for different constant values of the external loading (Fig. 1). Considering the specific kind of external forces, a static deflection can be defined with the help of the balance examination for the external and elastic forces - such an example illustrates Fig.2.

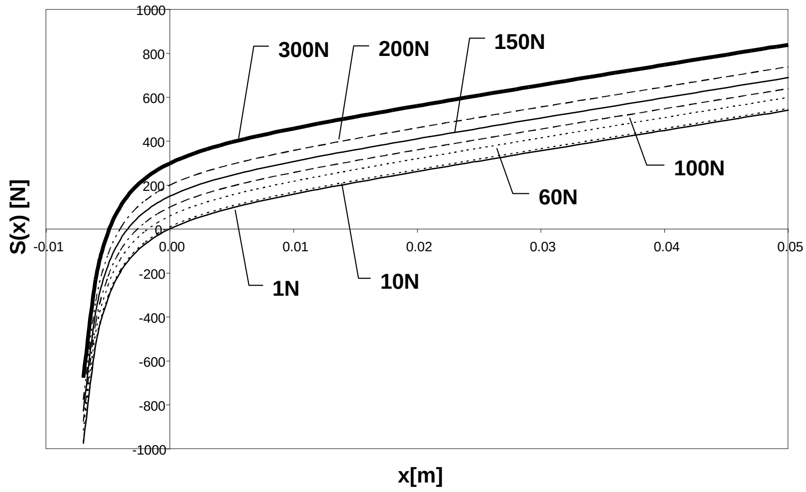


Fig.1. Elastic force $S(x)$ for various constant external loads

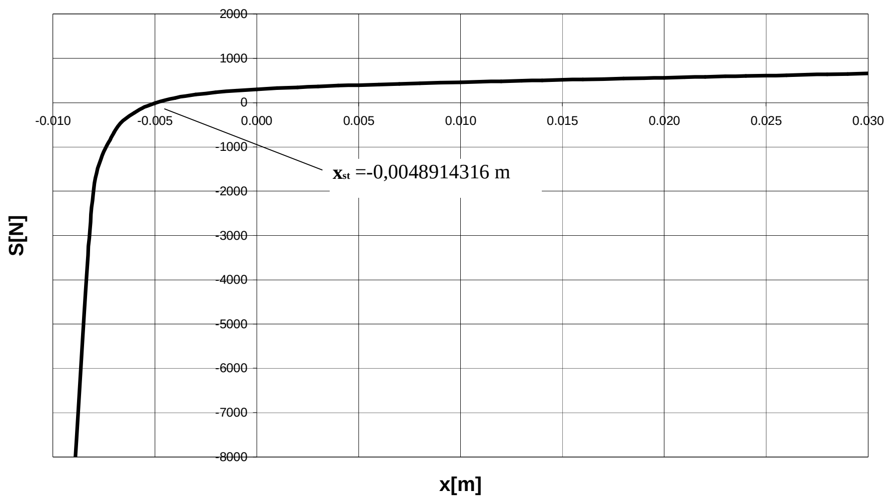


Fig.2. State of equilibrium of the external load F_0= 300N and the elastic force $S(x)$ – calculation of a static deflection

Irregularity of the obtained results of eq. (9) in the case of undamped forced vibration grows with an increasing of the external loads in resonant zones. An introductory statistical analysis of the obtained results (displacements) for different external loads, shows only the slight similarity to the one-side truncated Gaussian distribution (an example in Fig.3). The last mentioned property is a consequence of the one-side constraints by acting compressing forces on the system. When the difference between the real and theoretical normal distribution seems to be significant it may confirm that the signal is chaotic.

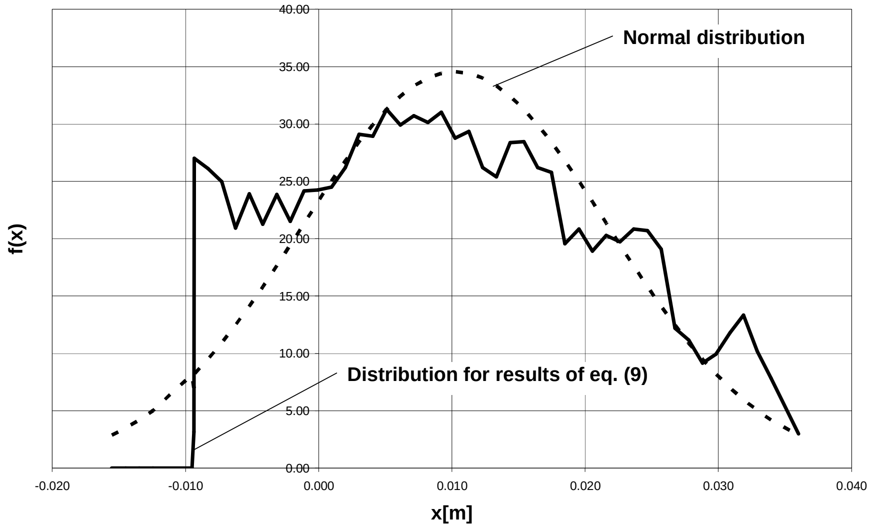
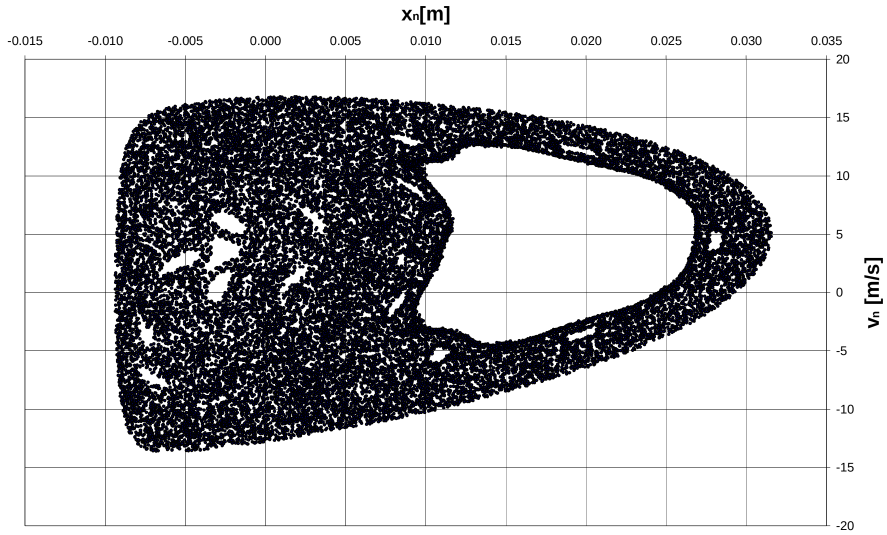


Fig.3. Comparison of the normal distribution between the real distribution of the obtained displacements

There have been conducted the computation to show how the results look in the phase-plain (x_n- displacement, v_n- velocity). By way of illustration, an example for an appropriate sampling period is included (Fig.4). In the conservatives systems there is known the fact that the phase volume does not change. On the basis of Poincaré reccurence theorem, it is possible to state that in such a case of systems, almost all trajectories pass nearby and close to their initial points. In opposite to the dissipative systems, there is lack of the attractive zones in the space phase - there is no constant points, limit cycles and strange attractors. However, there are also zones of a phase space in which chaos appears, and these zones are unattractive and existing alternately together with ones which have the regular evolution [Schuster H.G., 1988,]. As we can observe in Fig. 4a the phase-plain shown there is quite different from the next illustration presented in Fig.4b. The results illustrated on the phase-plain shown in Fig. 4a exhibit similarity to the chaotic vibrations and can be a sign that the motion is irregular. The next example shown in Fig.4b signalizes the periodicity of the vibrations. In these both examples the damping is excluded in the calculations and the data is only differ for the external loads.

As we have already mentioned, the elastomers have extremely low level of damping which can be neglected in practice. But now, for the sake of completeness we may also assume that dissipative forces play significant role. There are usually several of the control parameters for the system - in this case it is the external load, and when we introduce dissipative forces it is as well as the damping. For the same procedure of sampling as for that which has been used to create the phase plain, we obtain results in the form of bifurcation diagrams (Fig. 5).

(a)



(b)

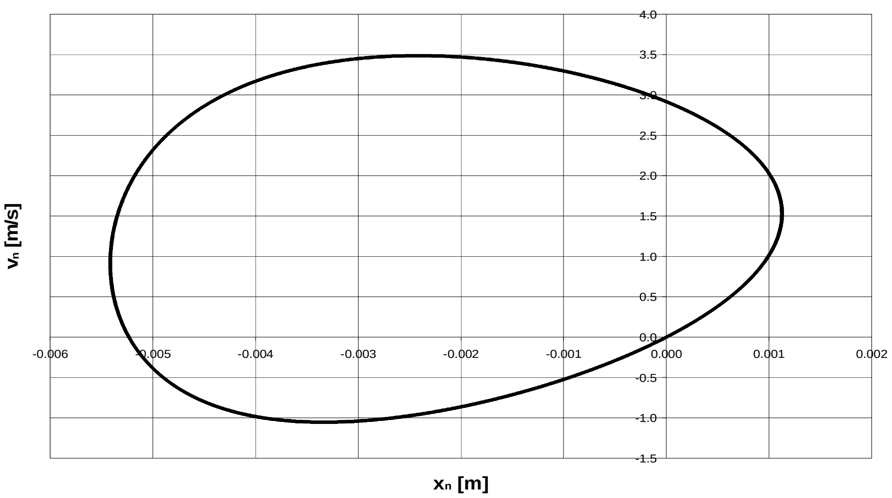


Fig.4. Poincaré portrait (x_n, v_n) for F=F_0=300N, ν/ω= 1 and 20000 points (a); F=F_0=100N, ν/ω= 1 and 20000 points (b)

It is obvious, that the damping level is a crucial factor which is influencing on the irregularity of the motion by acting the external loads of relatively considerable

values. This influence is also illustrated in the simulation of the responses to impulse forces in the case of an undamped system (Fig.6a) and for a rational level of damping (Fig.6b) - τ /T is a relation of a time duration of an impulse to the theoretical period of natural vibration.
The results of eq.(9) after performing the frequency analysis by use of the FFT (Fast Fourier Transform) for F=F_0=300 N and $\nu/\omega = 1$, are shown in Fig.7a. There is shown an influence of the long-term changes of the obtained time series in the lower and medium bands of frequency. On this background we can see the sub- and ultra-harmonics of the natural frequency of the system ω = 47,8Hz. In comparison with this spectrum, in Fig.7b there is shown the case of the spectrum for F=F_0=100N. We can notice that there is not any continuous background in the lower bands and we can observe only the discrete dominating values of harmonics of the natural frequency.

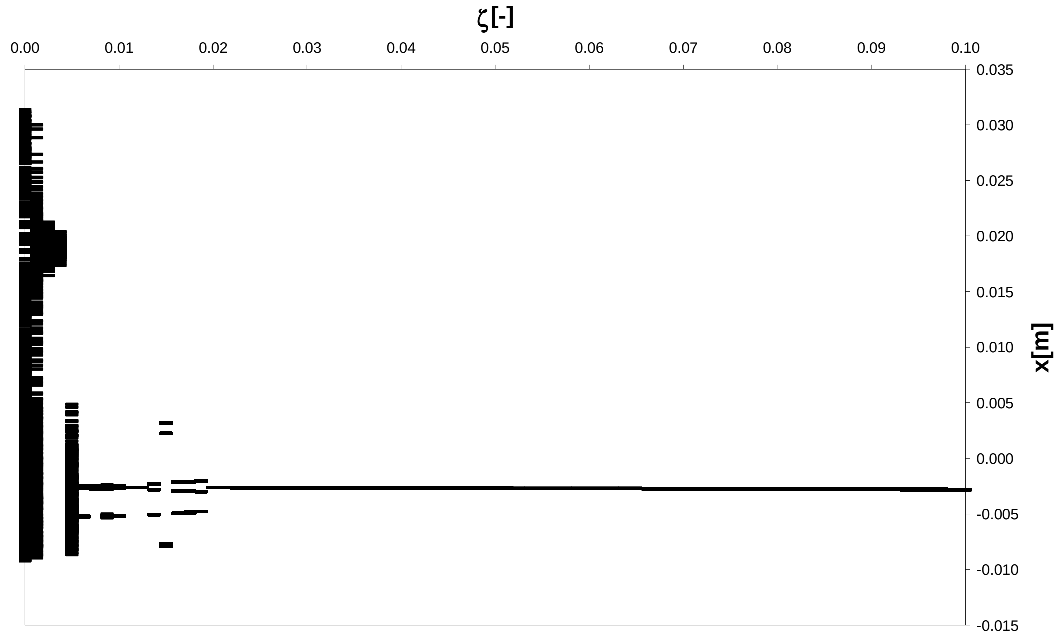
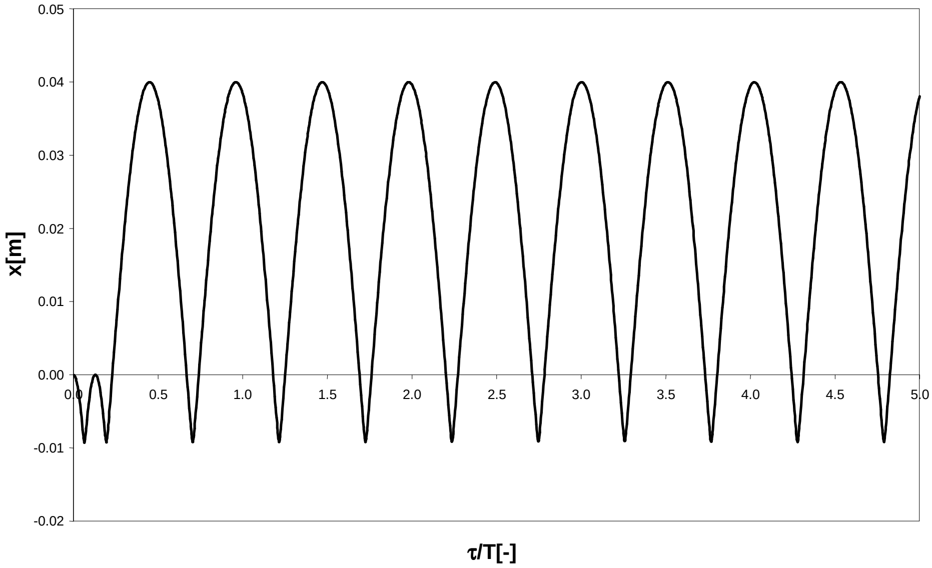


Fig.5. Bifurcation diagram for varied damping level by the constant external loads for F=F_0=300 N

(a)



(b)

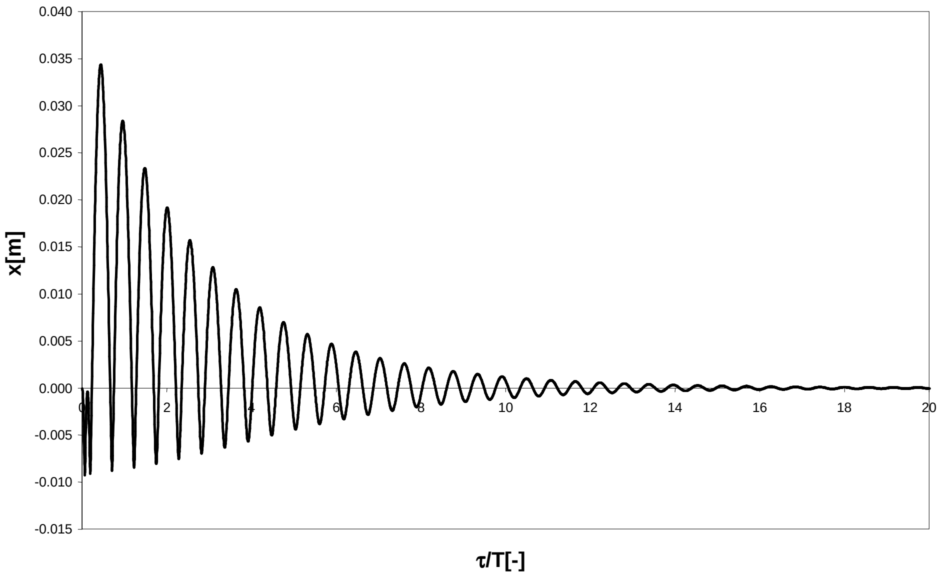
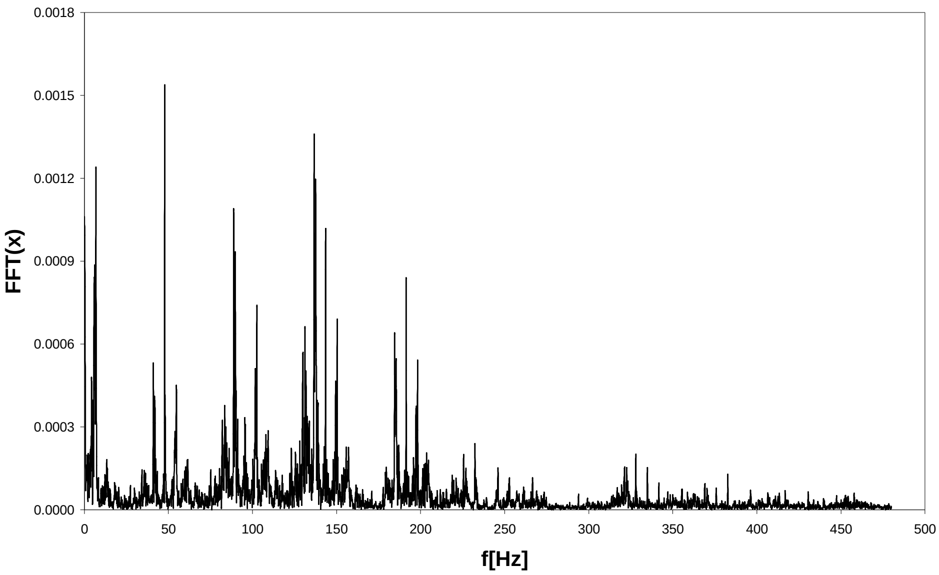
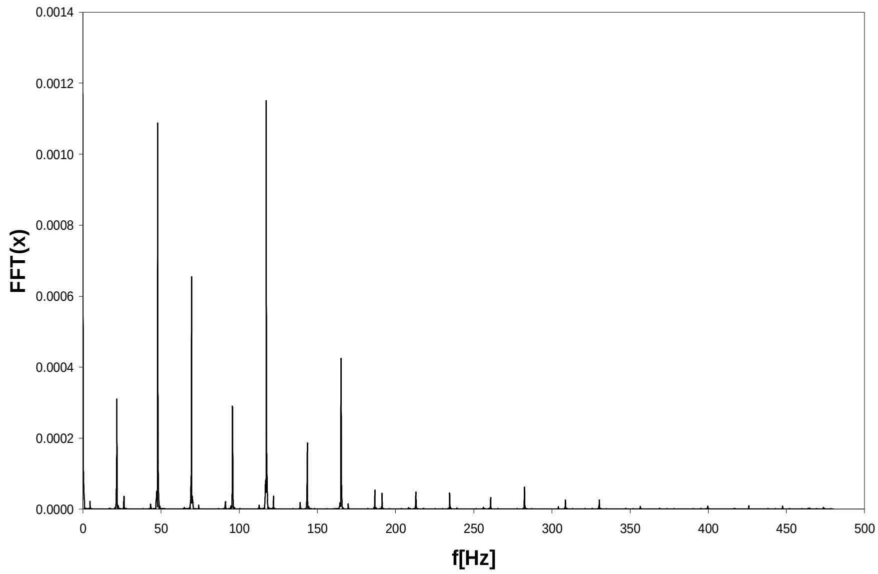


Fig.6. Simulation of the response to the impulse force for an undamped system τ/T = 0,2 and height of impulse F_0 = –1200 N (a), in case of damping factor ζ= 0,05; τ/T = 0,2 and height of impulse F_0 =–1200 N (b)

(a)



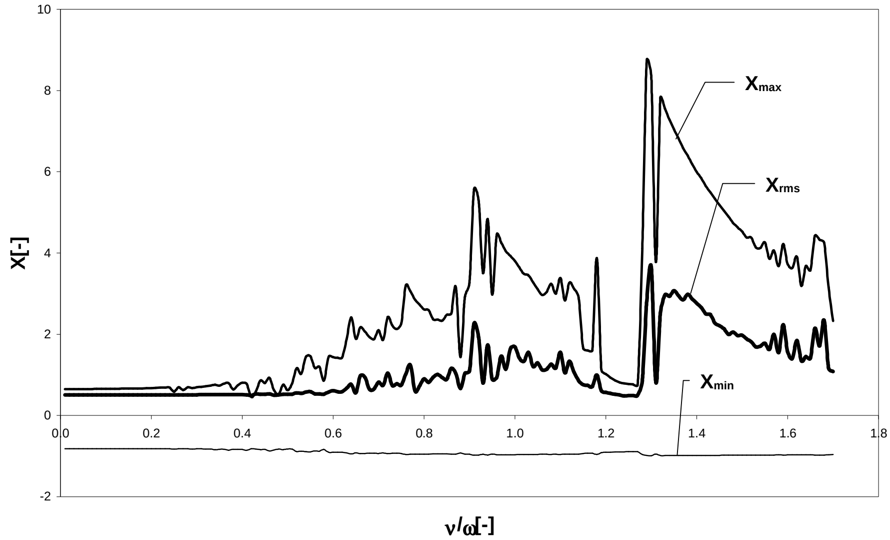
(b)



(b)

Fig.7. Frequency spectrum for displacements of the system (a) for F_0= F=300 N, ν/ω= 1 (b) for F_0= F=100 N, ν/ω= 1

Finally, (Fig. 8a and 8b) there are shown the vibration characteristics for the system subjected to the harmonic excitation vs. a non–dimensional relation of frequencies ν/ω. In these plots, values of maximal, minimal and root-mean square displacements (X_{max}, X_{min}, X_{rms}- respectively) are also non-dimensional after normalization by using the height of elastomer element *l*. (a)



(b)

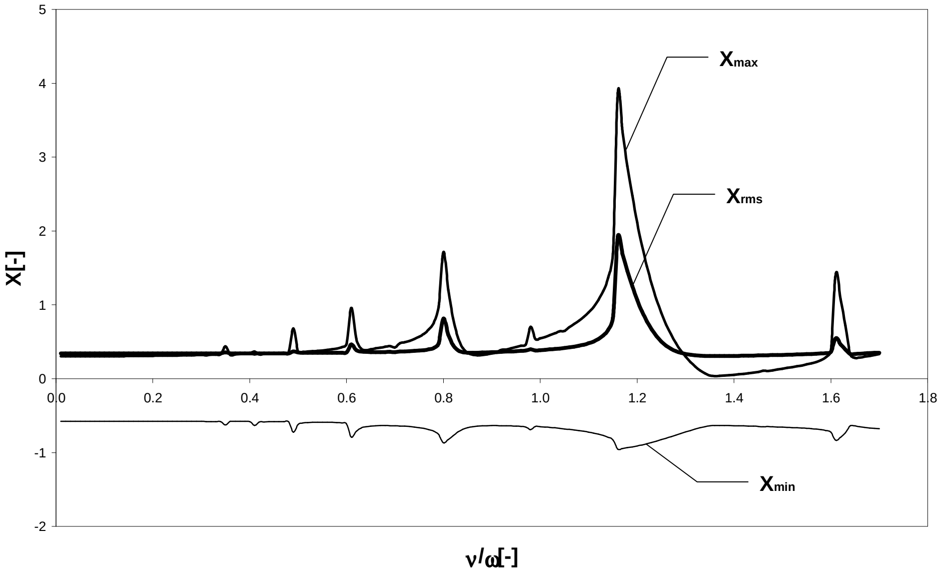


Fig.8. Vibration characteristic of non-dimensional displacements (X_{max},, X_{min}, X_{rms},) vs. dimensionless exciting frequencies for the constant initial values (a) F_0= F=300 N (b) F_0= F=100 N

The displacements have been obtained for the constant values of initial conditions and loads $F=F_0$=100N and 300N respectively. Again, as in the previous examples, the initial transient behaviour is not included, there have been only considered the results above time greater than 200*T* (*T* - period of excitation). Such kind of plots enables us to show dynamic behaviour and to predict the resonant zones and as well as to evaluate the possible danger of the failure by destructive vibration.

5. Conclusions

If the damping is neglected, irregular motion take place in the case of considerable loads and relatively high exciting frequencies in comparison with the natural frequency. In such cases the system exhibits dynamic behaviour similar to the chaotic motion. The permissible motion of the system is determined by its physical constraints. In one direction, the motion is limited theoretically, by structural damage as a result of compressing forces, and on the free side, by stretching of the elastomer element with height *l*.
From such a point of view for this kind of elastomers, because of a lack of detailed experimental damage characteristics for tension and compress forces for the present, there is significant difficulty for evaluation of material effects for varying with time and acting with high amplitudes external forces. The other imposed remark is, that the description of the damping in the model, may be more complicated then it having been considered. A negligible level of damping, is appropriate for the low load and low levels of frequencies of harmonic forces and /or for low and medium displacements. It means hypothetically, that for higher displacements or by the variable ambient temperature, significant dissipative forces act. It is worth remembering that the introduction of dissipative forces described by the equivalent viscous damping factor, changes significantly the forced vibration character of the system exited by the varying external loads.

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